

Scuola Superiore di Catania

Corso Specialistico

a.a. 2018-2019

The Structure of Choice Spaces

A choice space is a set of alternatives endowed with a contractive map, which associates to each menu a nonempty subset of selected items. We study choice spaces from an axiomatic perspective, with the ideal objective of obtaining a classification based on the satisfaction of structural properties. Although our analysis has a similar underlying rationale as the traditional theory of revealed preferences pioneered by Samuelson (1938), we depart from the classical approach insofar as we employ novel tools to determine the structure of an observed choice behavior.

Prerequisites. Elementary notions of set theory and propositional logic suffice. However, a certain degree of mathematical maturity is assumed. A basic knowledge of microeconomics, macroeconomics, general topology, combinatorics, graph theory, first-order logic, and algorithms may provide a better insight into some topics, but it is not strictly necessary.

Goal. The general objective of this course is to provide students with an overall comprehension of axiomatic choice theory from a novel point of view. The course is designed for students from Economics, Political Science, Mathematics and Computer Science, but theoretically also students from other departments may find the course suited to their needs. The analysis is structured in a large number of topics, which are all at the very frontier of the current research in the field. However, depending on students' response and interest, the lectures may be concentrate on selected topics.

References. There is a very large number of references. They are included not only to provide the students with the theoretical background for the course, but also (and especially) to suggest further readings on the topics discussed in class.

All references are listed at the end of this presentation, and they are labelled as either "B" (books), or "P" (published papers), or "W" (work in progress, possibly already submitted for publication). Students will be encouraged to pursue reading on related topics, according to their interests.

Syllabus

1. Preliminaries [B1, B2, B4, B5]

- (a) Preferences
- (b) Utilities
- (c) Choices
- (d) Elements of logic

2. Choice Rationalization [B3, P1, P7, P10]

- (a) Revealed preference theory
- (b) $(m; n)$ -rationalizability
- (c) $(m; n)$ -Ferrers properties and money-pumping phenomena

3. Simultaneous Multi-Rationalizability [P11, W6]

- (a) Rationalization by multiple rationales
- (b) Free multi-rationalization
- (c) Monotonic multi-rationalization
- (d) Other types of multi-rationalization (transitive, listable, etc.)

4. Sequential Multi-Rationalizability [P3, P12, W6]

- (a) Rational shortlist method and n -sequential rationalizability
- (b) Toward a characterization

5. Democratic Choices [W1]

6. Congruences in Choice Spaces [W2, W5]

- (a) Definition and characterization
- (b) Reactivity and consistency
- (c) Revealed indiscernibility

7. Resolutions in Choice Spaces [W3, W4]

- (a) Definition and examples

- (b) Consistency
- (c) Structural results
- (d) Iterated and multiple resolutions

8. Decision Procedures in Choice Spaces [P2]

9. Appendix

- (a) Bi-preferences [W7]
- (b) NaP-preferences [P4, P5, P6, P8, P9]
- (c) Schmeidler's theorem [P13, W7]

References

B1 Aleskerov, F., D. Bouyssou, and B. Monjardet, B. (2007). *Utility Maximization, Choice, and Preference* (2nd Ed.). Springer, Berlin.

B2 Bridges, D., and G. B. Mehta (1995). *Representations of Preference Orderings*. Lecture Notes in Economics and Mathematical Systems 422, Springer, Berlin.

B3 Chambers, C.P., and F. Echenique (2016). *Revealed Preference Theory* (Econometric Society Monographs). Cambridge University Press.

B4 Fitting, M. (1996). *First-order Logic and Automated Theorem Proving* (2nd Ed.). Springer-Verlag, New York.

B5 Schwartz, J. T., D. Cantone, D., and E. G. Omodeo (2011). *Computational Logic and Set Theory: Applying Formalized Logic to Analysis*. Springer-Verlag, Berlin.

P1 Cantone, D., A. Giarlotta, S. Greco, and S. Watson (2016). (m; n)-rationalizable choices. *Journal of Mathematical Psychology* 73: 12-27.

P2 Cantone, D., A. Giarlotta, and S. Watson (2017). The satisfiability for Boolean set theory with a choice correspondence. *Proceedings of the Eight International Symposium on Games, Automata, Logics and Formal Verification (GandALF 2017)*, Rome, Italy, 20-22 September.

P3 García-Sanz, M., and J. C. R. Alcantud (2015). Sequential rationalization of multivalued choice. *Mathematical Social Science* 74: 29-33.

P4 Giarlotta, A. (2014). A genesis of interval orders and semiorders: transitive NaP-preferences. *Order* 31: 239-258.

P5 Giarlotta, A. (2015). Normalized and strict NaP-preferences. *Journal of Mathematical Psychology* 66: 34-40

P6 Giarlotta, A., and S. Greco (2013). Necessary and possible preference structures. *Journal of Mathematical Economics* 42(1): 163-172.

P7 Giarlotta, A., and S. Watson (2014). The pseudo-transitivity of preference relations: strict and weak $(m; n)$ -Ferrers properties. *Journal of Mathematical Psychology* 58: 45-54.

P8 Giarlotta, A., and S. Watson (2017). Well-graded families of NaP-preferences. *Journal of Mathematical Psychology* 77: 21-28.

P9 Giarlotta, A., and S. Watson (2017). Necessary and possible indifferences. *Journal of Mathematical Psychology* 81: 98-109.

P10 Giarlotta, A., and S. Watson (2018). Strict $(m; 1)$ -Ferrers properties. *Journal of Mathematical Psychology* 82: 84-96.

P11 Kalai, G., A. Rubinstein, A., and R. Spiegel (2002). Rationalizing choice functions by multiple rationales. *Econometrica* 70/6, 2481-2488.

P12 Manzini, P., and M. Mariotti (2007). Sequentially rationalizable choice. *American Economic Review* 97: 1824-1839.

P13 Schmeidler, D. (1971). A condition for the completeness of partial preference relations. *Econometrica* 39/2: 403-404.

W1 Alcantud, J. C. R., A. Giarlotta, and S. Watson. Democratic choices.

W2 Cantone, D., A. Giarlotta, and S. Watson. Congruence relations on choice spaces.

W3 Cantone, D., A. Giarlotta, and S. Watson. The structure of choices: Resolutions I.

W4 Cantone, D., A. Giarlotta, and S. Watson. The structure of choices: Resolutions II.

W5 Cantone, D., A. Giarlotta, and S. Watson. On the preservation of formulae by congruence relations in a choice space.

W6 Cantone, D., A. Giarlotta, and S. Watson. Multi-rationalizability.

W7 Giarlotta, A., and S. Watson. A theory of bi-preferences and some extensions of Schmeidler's theorem.